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A parametric study of a drive by bridge inspection system based on the Morlet wavelet

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Abstract. Many of the bridges currently in use worldwide are approaching the end of their design lives. However, rehabilitating and extending the lives of these structures raises important safety issues. There is also a need for increased monitoring which has considerable cost implications for bridge management systems. Existing structural health monitoring (SHM) techniques include vibration-based approaches which typically involve direct instrumentation of the bridge and are important as they can indicate the deterioration of the bridge condition. However, they can be labour intensive and expensive. In the past decade, alternative *indirect* vibration-based approaches which utilise the response of a vehicle passing over a bridge have been developed. This paper investigates such an approach; a low-cost approach for the monitoring of bridge structures which consists of the use of a vehicle fitted with accelerometers on its axles. The approach aims to detect damage in the bridge while obviating the need for direct instrumentation of the bridge. Here, the effectiveness of the approach in detecting damage in a bridge is investigated using a simplified vehicle-bridge interaction (VBI) model in theoretical simulations and a scaled VBI model in a laboratory experiment.

In order to identify the existence and location of damage, the vehicle accelerations are recorded and processed using a continuous Morlet wavelet transform and a damage index is established. A parametric study is carried out to investigate the effect of parameters such as the bridge span length, vehicle speed, vehicle mass, damage level and road surface roughness on the accuracy of results.

Introduction

Bridges are an integral part of transport networks worldwide. Due to the ageing of existing bridge stocks, it is necessary for the relevant authorities to establish preventative maintenance strategies to extend the safe service lives of those bridges in use which have already exceeded or are approaching their design life. Hence, over the past two decades there has been a focus on the development of effective techniques for the monitoring of the condition of structures such as bridges. These structural health monitoring (SHM) techniques [1] are generally vibration based and require measurement and data acquisition equipment to be installed directly on the bridge, which can be effective but labour intensive and expensive. These approaches are arguably becoming a more critical part of maintenance strategies. More recently, there has been a move towards the development of *indirect* vibration-based approaches utilising the response of a vehicle passing over a bridge. This type of approach aims to reduce or eliminate the need for direct instrumentation of the bridge thus providing a more efficient and low-cost alternative. This paper investigates a wavelet-based indirect approach for the periodic monitoring of bridge structures which consists of the use of a vehicle fitted with accelerometers on its axles. The aim of the approach is to utilise the vehicle response to detect changes in the bridge corresponding to changes in the structural condition, i.e., damage. In this paper, the effectiveness of this approach is investigated both theoretically and experimentally.

The use of the indirect approach to identify bridge properties from the vehicle response has been investigated by many researchers. The feasibility of extracting properties such as the bridge's natural frequency and changes in damping from the response of a passing vehicle has been verified in theoretical simulations by analysing the spectra of accelerations [2-5]. Experimental investigations have been conducted to examine the feasibility of such an approach as part of a drive-by inspection

system for bridge monitoring [6,7] in which the bridge frequency and changes in damping are extracted from the vehicle response. Field trials have taken place to investigate indirect monitoring methods for bridges [8,9]. It is found that accurate determination of the bridge frequency is feasible at low speeds and when there is sufficient dynamic excitation of the bridge due to the road roughness having a greater influence than the bridge on the vehicle response. Further bridge condition assessment techniques utilising vehicle accelerations focus on the detection of damage via identification of bridge stiffness and damping using novel numerical approaches [10,11]. Both of these studies find that signal noise, road surface roughness and errors in the assumed numerical models do not have a significant effect on the accuracy of the respective approaches.

The popularity of wavelet theory and in particular, the use of wavelets in techniques to identify structural damage, has risen considerably in recent years as it allows a signal to be analysed in both time and frequency domains simultaneously. Examples can be found in [12] which demonstrate the capacity of the wavelet transform to capture time-frequency information and a discussion of the use of wavelet analysis in structural health monitoring applications can be found in [13]. Their potential to be used in statistical pattern recognition approaches for damage detection is highlighted in [14], in which a damage sensitive feature (DSF), based on the continuous Haar wavelet transform of a vibration signal, is applied to accelerations of the ASCE Benchmark Structure under different damage conditions. The DSF is defined as the energy of the wavelet coefficients calculated at particular scales and damage is identified by comparing the mean DSF for undamaged and damaged states.

The use of wavelets has also been extended to indirect approaches for the purpose of bridge damage detection; an approach is numerically investigated in [15] which aims to identify the existence and location of cracks in a bridge from the vehicle displacement response by using the Symlet wavelet transform. A cracked finite element (FE) beam model and 4 degree-of-freedom (DOF) half-car vehicle model are used in simulations and no road profile is included. Peaks at particular scales are observed in the wavelet transform of the vehicle displacement response when it passes over cracks and crack depths of up to 10% are detected. It is found that it is easier to detect cracks at lower speeds and when they are deeper. The effect of white noise on crack detection is investigated and for 6% noise, a 50% crack depth is detected at 2 m/s.

A numerical investigation is also carried out by [16] in which a very simple vehicle-bridge interaction (VBI) model is used to compare both direct and indirect methods which utilise a Gaussian 4 wavelet to identify the existence and location of cracks in beams. The continuous wavelet transforms (CWTs) of beam and vehicle displacements are used to identify cracks modelled as rotational springs connecting elements. The indirect method is found to be more effective than the direct method and cracks with a depth of more than 10% of beam depth are detected. The authors develop a damage index which has an explicit expression and identifies crack depth and location.

In this paper, the aim is to investigate the effectiveness of wavelet based indirect approach for bridge monitoring using vehicle accelerations. Firstly, in theoretical simulations, a simplified VBI simulation model is created in MATLAB and is used to investigate the effectiveness of the approach in detecting damage in a bridge. A time-frequency analysis is carried out in order to identify the existence and location of damage from the vehicle accelerations. For this purpose, the accelerations are processed using the Morlet CWT. A basic damage index is developed based on this analysis. In a parametric study the bridge span length, vehicle speed, vehicle mass, road roughness, damage level and location are varied in simulations to investigate the effect on the accuracy of results. Secondly, in the laboratory, a scaled VBI model is used in experiments to validate the theoretical results.

Methodology

Theoretical model. A coupled VBI model described in detail in [17] is used in theoretical simulations (Fig. 1) with a sampling frequency of 100 Hz and the solution is obtained at each time step using the Wilson-Theta direct integration scheme [18], using the optimal value of $\theta = 1.421$ for unconditional stability. Similar models which take account of coupling between the vehicle and bridge can be found in a comprehensive review by González [19]. The vehicle is represented by a 2

DOF half-car which crosses the bridge model at constant speed c (Fig. 1). The two DOFs correspond to sprung mass bounce displacement, y_s , and sprung mass pitch rotation, θ_s . The vehicle body and axle component masses are represented by the sprung mass, $m_s = 18\,000$ kg. A combination of springs of linear stiffness $K_1 = 2 \times 10^6$ N/m and $K_2 = 5 \times 10^6$ N/m and viscous dampers with damping coefficient $C_1 = 10 \times 10^3$ N s/m and $C_2 = 20 \times 10^3$ N s/m represent the suspension components for the front and rear axles. Also, $I_s = 103,840$ kg m² is the sprung mass moment of inertia and the distance of each axle to the vehicle's centre of gravity (o) is given by $D_i = 2.375$ m ($i = 1, 2$). The vehicle has both bounce and pitch frequencies of $f_{v,1} = 2.36$ Hz and $f_{v,2} = 3.73$ Hz respectively. The equations of motion of the vehicle are obtained by imposing equilibrium of all forces and moments acting on the vehicle and expressing them in terms of the degrees of freedom. Sprung mass acceleration measurements are recorded above the suspension of each axle in simulations (Fig. 1) and the relationship between the degrees of freedom of the vehicle and the measurements is defined by the following equation,

$$\ddot{y}_{s,i} = \ddot{y}_s - (-1)^i D_i \ddot{\theta}_s \quad (i = 1, 2). \quad (1)$$

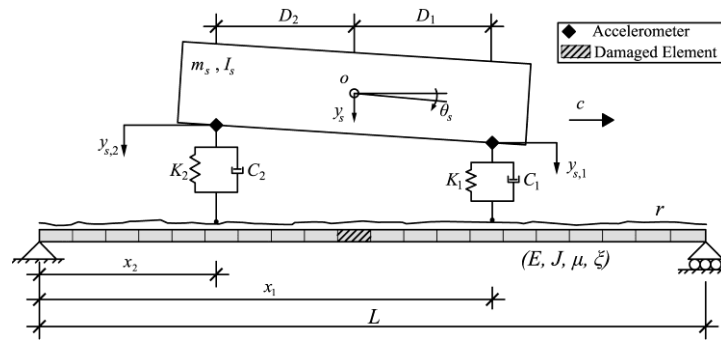


Fig. 1 Vehicle-bridge interaction model

The bridge is represented by a simply supported FE beam of total span length L . It consists of 64 discretised beam elements with 4 degrees of freedom which have constant mass per unit length, μ , modulus of elasticity E and second moment of area J . It follows that the beam element stiffness is the product of E and J , denoted EJ . Damage is applied to the beam via percentage reduction of the stiffness of individual elements, corresponding to localized damage within the bridge. Rayleigh damping is adopted for the beam whereby the viscous damping ratio $\zeta = 3\%$ is assumed to be the same for the first two modes [20]. The properties of the three bridge spans used in simulations are given in Table 1.

Table 1 Finite element beam properties

Span Length, L [m]	Intact Element Stiffness, EJ [N m ²]	Mass per unit length, μ [kg/m]	1st natural frequency of vibration, $f_{b,1}$ [Hz]
15	1.846×10^{10}	28 125	5.66
25	4.865×10^{10}	18 358	4.09
35	1.196×10^{11}	21 752	3.01

Experimental model. A scaled two-axle vehicle with axle spacing of 0.4 m was fitted with 2 accelerometers at the centre of each axle to monitor bounce motion (Fig. 2(a)). A wireless router and data logger allowed accelerations to be recorded remotely. The vehicle's speed was maintained constant by an electronic controller; each bridge crossing was repeated 5 times. Its entry and exit to the beam was monitored using strain sensors. Three speeds were adopted; $S1 = 0.93$ m/s, $S2 = 1.16$ m/s and $S3 = 1.63$ m/s. Two vehicles were used, V1 and V2, of masses 21.6 kg and 25.8 kg respectively. Both had bounce frequencies of 2.93 Hz while pitch frequencies were 3.9 Hz and 3.7 Hz respectively. The scaled bridge model was a simply supported steel beam (Fig. 2(b)) with span, $L_{exp} = 5.4$ m and incorporated a scaled road surface profile. It had frequency $f_{b,exp} = 2.6$ Hz, mass per unit length, $\mu_{exp} = 52$ kg/m and stiffness, $EJ_{exp} = 120,700$ N m². It was fitted with accelerometers and displacement transducers at mid-span and quarter points to measure its response during vehicle

crossings. For the experiment, damage was applied via 0.7 m long rectangular saw-cuts in the beam's flanges between midspan and $3/8^{\text{th}}$ of the span. Four scenarios were investigated: Intact, D1, D2 and D3 corresponding to no damage, 5 mm, 10 mm and 15 mm cuts respectively (Fig. 2 (c)). A sampling frequency of 100 Hz was used in the experiment.

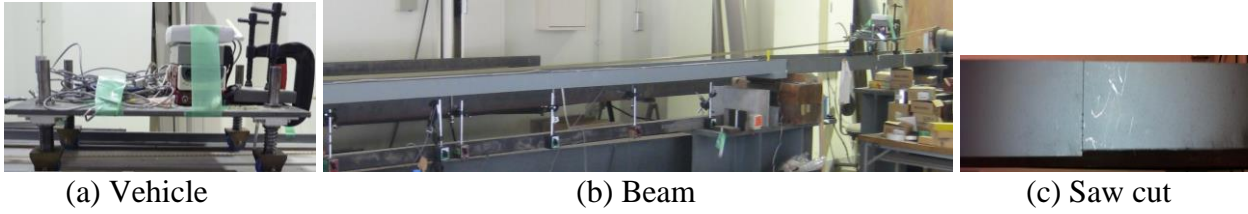


Fig. 2 Experimental setup

Morlet Continuous Wavelet Transform. The continuous wavelet transform [21] of a function $f(t) \in L^2(\mathbf{R})$ is given as

$$Wf(a, b) = \int_{-\infty}^{\infty} f(t) \frac{1}{\sqrt{a}} \psi^* \left(\frac{t-b}{a} \right) dt \quad (2)$$

where * indicates the complex conjugate of the mother wavelet function, $\psi(t) \in L^2(\mathbf{R})$, shown in Eq. 3,

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}} \psi \left(\frac{t-b}{a} \right). \quad (3)$$

The wavelet function is scaled by a and translated by b . The mother wavelet adopted for this investigation is the Morlet wavelet, described by Eq. 4. It is a real valued symmetrical wavelet and is selected for this analysis based on a preliminary study of the performance of a number of mother wavelets including Mexican Hat and Gaussian. Time localisation is an important criterion for this approach in order to detect the damage location and it is found that the Morlet wavelet provides the best balance between time and frequency resolution for the approach presented in this paper. Therefore the Morlet CWT of vehicle acceleration responses is adopted as a damage sensitive feature.

$$\psi(t) = e^{-\frac{t^2}{2}} \cos(5t). \quad (4)$$

Damage Index. The CWTs of accelerations obtained in theoretical simulations and the experiment are analysed in both time and frequency domains simultaneously for the purpose of damage detection. All acceleration signals are normalised using their standard deviations before applying the CWT. Peaks occurring in the *difference* between wavelet coefficients from healthy and damaged cases indicate the existence and location of damage. A damage index based on the maximum magnitude of these peaks at particular frequencies is established. It focuses on maximum values at frequencies related to the vehicle as they are found to be the most dominant in the VBI.

Results and Discussion

Theoretical Scenarios. The aim in theoretical simulations is to investigate the effectiveness of the wavelet based indirect approach in identifying the existence and location of damage in a bridge for a range of parameters. For this purpose, bridge span lengths of 15 m, 25 m and 35 m, vehicle speeds of 2 m/s, 5m/s, 10 m/s and 20 m/s, vehicle masses of 9 tonnes and 18 tonnes, smooth and a range of rough road profiles are tested in simulations. The severity and location of the damage are also varied; stiffness reductions from 5% up to 20% are applied to a beam element at either $L/2$ or $5L/8$.

Effect of Bridge Span Length. Fig. 3 illustrates the effect of bridge span length on the ability of this approach to detect damage. In each case, the differences between wavelet coefficients of axle

accelerations obtained for the healthy and the 5% damage case are shown. Here, the damage is detected in all cases but is only located in Fig. 3(a) and (b) at peaks in the vehicle frequency response of 2.36 Hz; the vertical lines represent entry and exit of the axle on the damaged beam element at $L/2$. It can be seen that as the span increases, the overall maximum magnitudes increase and time localisation improves, however, the bridge frequency becomes closer to the vehicle frequency also, reducing the dominance of the damage peaks detected at the vehicle frequency. This suggests it may be beneficial to select a vehicle with frequencies which are not close to the bridge frequency.

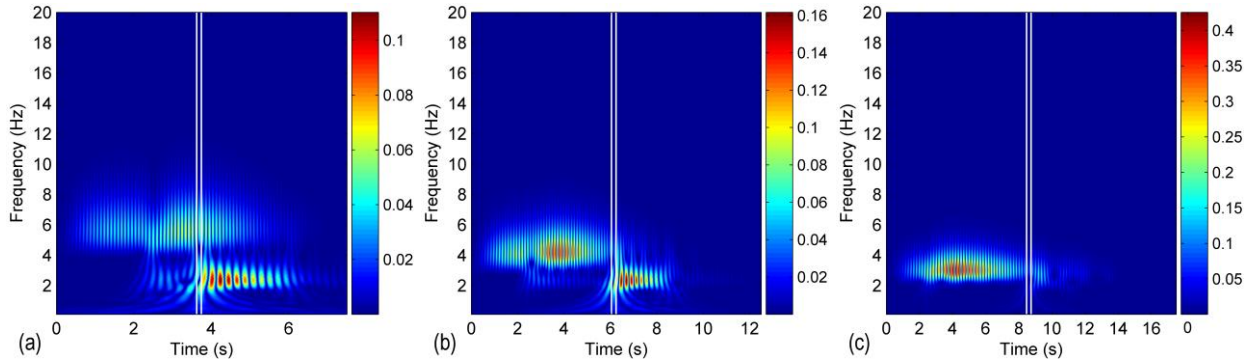


Fig. 3 Difference between wavelet coefficients of axle 1 accelerations on (a) 15 m span (b) 25 m span (c) 35 m span; speed is 2 m/s, damage level 5% at $L/2$, smooth road profile.

Effect of Vehicle Speed. It is shown in Fig. 4 that compared to Fig. 3(a) for 2 m/s, as the vehicle speed increases, the response time history shortens and the resolution decreases therefore it becomes more difficult to locate damage although it can be detected for all speeds tested. This indicates that lower vehicle speeds are best for locating damage using this approach.

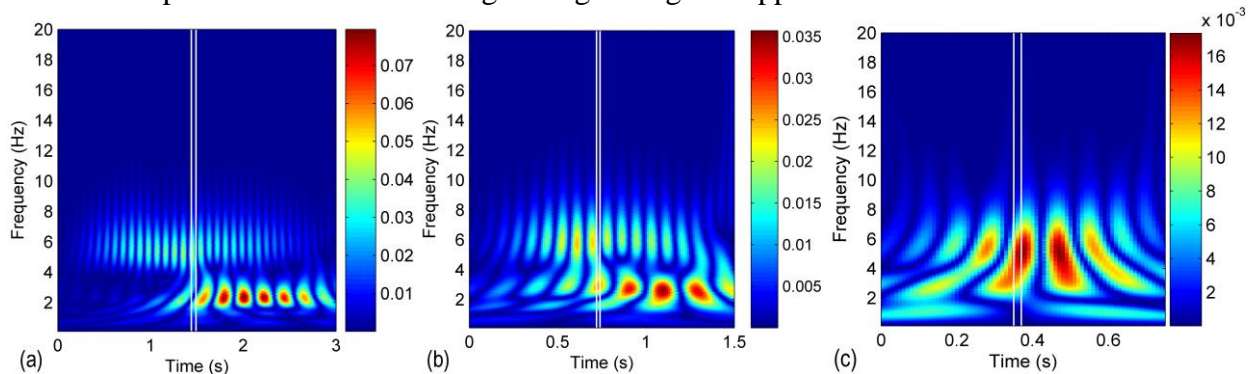


Fig. 4 Difference between wavelet coefficients of axle 1 accelerations for 15 m span (a) 5 m/s (b) 10 m/s (c) 20 m/s; damage level 5% at $L/2$, smooth road profile.

Effect of Vehicle Mass and Damage Level and Location. To test the effect of varying vehicle mass, a vehicle of total mass 9 t is used in simulations. As the vehicle frequency is found to be an important parameter for damage localisation, its axle properties are selected in order to assign the same bounce frequency as the 18 t vehicle. An example of results are shown in Fig. 5(a) and it can be seen by comparing with Fig. 3(a) that damage detection has improved slightly for the 9 t vehicle, indicating that the approach may not be limited to particular vehicle masses.

Thus far, results for 5% damage applied at bridge midspan have been presented; Fig. 5(b) presents an example of results obtained for a higher damage level of 10% at $L/2$ for the 18 t vehicle. Comparing Fig. 3(a) and Fig. 5(b), it is observed that for an increased damage level, the magnitudes of coefficients increase also. A similar trend occurs for higher damage levels thus it may be possible to quantify damage using this relationship. Fig. 5(c) shows the wavelet coefficients corresponding to the scenario represented in Fig. 3(a), except now considering the damaged element location at $5L/8$. Comparing Fig. 5(c) to Fig. 3(a), the coefficient magnitude increases slightly at the vehicle response while the location accuracy is similar, indicating that this approach can identify the damage regardless of its position along the bridge span. Similar results are obtained for higher damage levels at $5L/8$.

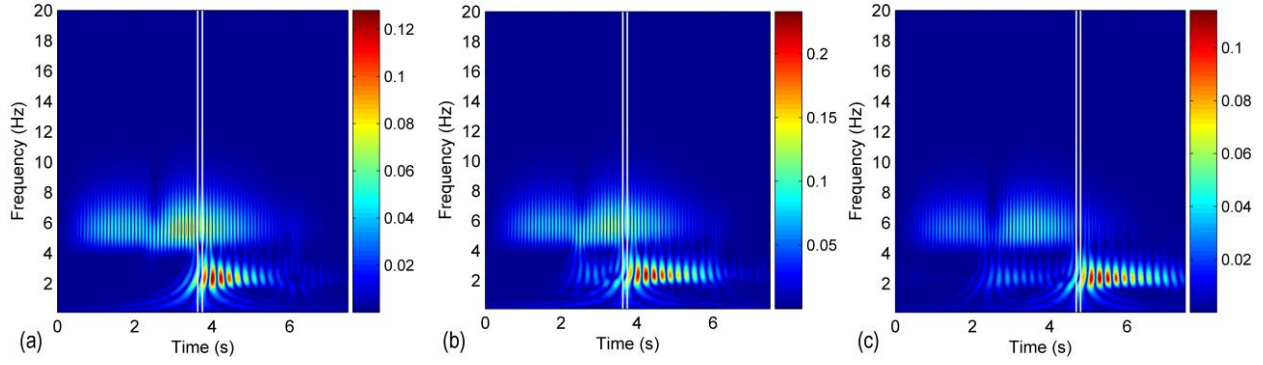


Fig. 5 Difference between wavelet coefficients of axle 1 accelerations for 15 m span (a) 9 t vehicle, 5% damage at $L/2$ (b) 18 t, 10% damage at $L/2$ (c) 18 t, 5% damage at $5L/8$; speed is 2 m/s.

The Effect of Road Roughness. Results presented thus far were for a smooth road profile. However, as past studies encountered difficulties in detecting changes in the bridge response from the vehicle response in the presence of rough road profiles, here the sensitivity of the approach to road roughness is tested for a range of 50 ISO class ‘A’ (very good) road profiles [22] in simulations. An example of results corresponding to the 15 m bridge span and a speed of 2 m/s are presented in Fig. 6(a). The vehicle response dominates in the region of 2.36 Hz and it is still possible to detect damage in the bridge. However, compared to Fig. 3(a) for a smooth profile, locating the damage accurately is more difficult for the rough profile. To illustrate the effectiveness of the approach for rough road profiles, a damage index based on the maximum coefficient difference in the frequency range from 0.5 Hz to $(f_{v,1} + f_{b,1})/2$ is calculated and plotted for all speeds and profiles tested in Fig. 6(b). To allow comparison between indices for all damage levels, they are standardized for each speed using their means and standard deviations; indices for all speeds are grouped according to midspan damage level.

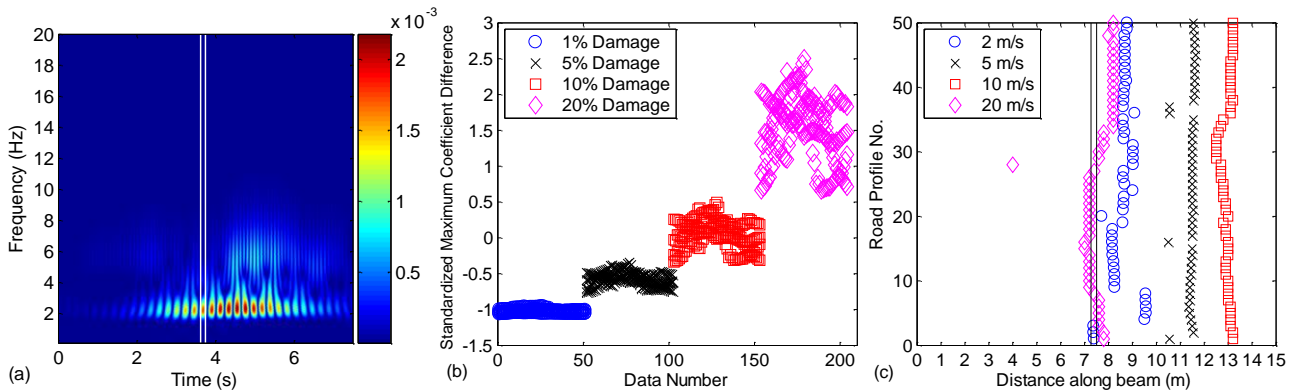


Fig. 6 (a) Difference between wavelet coefficients of axle 1 accelerations 5% damage for class A road profile; (b) Identified damage indices and (c) locations for 15 m span, 50 class A profiles.

A damage level of 1% is included here for comparison. It can be seen that as the damage level increases, the damage index also increases. This indicates that the index could be effective for the detection and quantification of damage. Also, it is not significantly affected by speed. Fig. 6(c) summarizes the identification of damage location; damage level was found to have no effect hence only one level is represented here. The mean errors as percentages of the bridge span length are 7.6%, 26.2%, 36.4% and 3.2% for 2, 5, 10 and 20 m/s respectively. Similar to earlier observations, it is found that as speed increases, the ability to locate damage generally decreases due to the decrease in VBI time. However, for 20 m/s, the accuracy appears to be the best here which may be due to higher excitation of the bridge (Fig. 4(c)) but considering the poor resolution at this speed, it is expected that accuracy would decrease if the speed was increased further.

Experimental scenarios. The results of the application of the approach to the scaled experiment are summarized here for axle 2 of the vehicle, which was found to be more accurate due to its frequency. Fig. 7(a) shows an example of the difference between wavelet coefficients of accelerations obtained for the Intact and D3 damage scenarios. The largest peaks occur at the start of the time history at the

vehicle frequency of 3.7 Hz however the damage location is detected at the vehicle and bridge frequencies. Fig. 7(b) and (c) show the indices calculated for all damage scenarios, speeds and tests for both vehicles. It is clear that the index is not as sensitive here as in simulations; D2 is difficult to distinguish from D1 although D3 can be distinguished more easily. Speed is not found to have a significant effect.

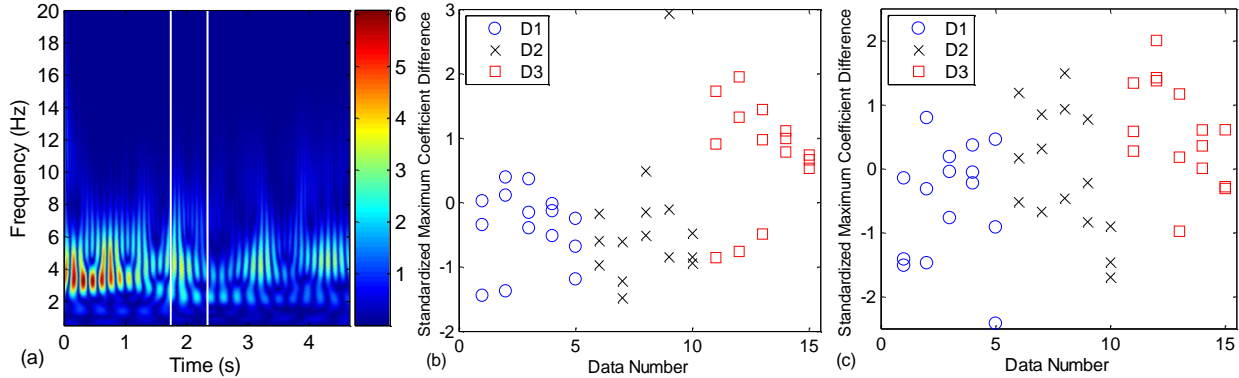


Fig. 7 Experiment results (a) Difference between wavelet coefficients of accelerations above axle 2 for vehicle V1 and speed S2; Identified damage indices for (b) V1 and (c) V2 for all speeds and tests.

Conclusions

This paper investigates the feasibility of an alternative wavelet-based approach for the periodic monitoring of bridge structures consisting of the use of a vehicle instrumented with accelerometers on its axles. In theoretical simulations for smooth and rough road profiles, damage applied via beam element stiffness reductions is detected at the vehicle frequency response in the difference between healthy and damaged wavelet coefficients of accelerations. It is found that the approach can locate the damage more accurately for lower speeds and also for longer bridge spans provided vehicle and bridge frequencies are not close. It is found that vehicle mass is not as significant as vehicle frequency for this approach. The damage level is indicated by the maximum wavelet coefficient difference magnitude hence a damage index is established which allows damage levels to be detected and distinguished from each other. However, in a laboratory experiment damage can be detected but it is found to be more difficult to distinguish between damage scenarios using this index. Overall, this paper has illustrated the potential of this low-cost approach to be used as a bridge monitoring tool and highlighted conditions within which it can detect bridge damage with reasonable accuracy. Further work is required to address challenges associated with the real-world application of this approach.

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